

3- and 4- body meson- nuclear clusters

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Abstract

The binding energies and matter distributions for the 3- body systems like ϕ - meson $+2N$, $2\phi + N$ and 4- body system like $\phi + 3n$ are calculated. For the 3- particle systems two- dimensional Faddeev equations in the differential form are used. For the 4- body system $\phi + 3n$ the folding model is applied.

1 Introduction

As it has recently been intensively discussed [1-7], there are indications of strong attraction of mesons with one strange quark K^- (\bar{K}) to few- nucleon nuclei. Along this line, it is interesting to look at the interaction of a meson with two strange quarks like the ϕ -meson with light nuclei. Already existing theoretical investigations of the ϕ -meson interaction show rather strong attraction between a ϕ -meson and a nucleon. Indeed, the calculation of the $\phi - N$ interaction within the quark model [8] and on the basis of a totally different phenomenological model [9] based on the dominant role of the $s\bar{s}$ configuration in the ϕ -meson structure predicts considerable $\phi - N$ attraction with a binding energy of about 9 MeV for the ϕN system.

Strong attraction is not very surprising in view of the physical arguments that the strong $K^- N$ attraction originates from the influence of subthreshold resonances Λ_{1405} and Σ_{1385} .

Indeed, let us compare the mass of the state $\phi + N$ with the masses of two subthreshold states $K + \Lambda_{1405}$ and $K + \Sigma_{1385}$. It turned out that distances of the above subthreshold states from the $\phi + N$ threshold are of the same order of magnitude as in the $K^- N$ case, which means that as in $K^- N$ one can expect strong influence of Λ_{1405} and Σ_{1385} and strong attraction also in the ϕN system.

Bearing in mind this sort of strong attraction in the ϕN system, it is interesting to consider the possibility of bound states of a ϕ -meson with a few nucleons, in particular, with two neutrons or two protons. This is in fact a question concerning the existence of new nuclear clusters which do not exist

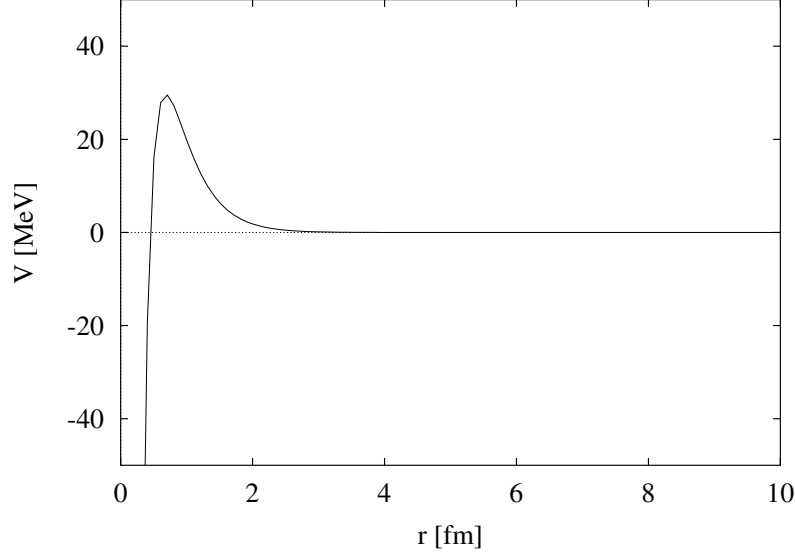


Figure 1: The $\phi\phi$ local potential

without a ϕ - meson. In what follows, we will calculate the binding energies of the three-body systems ϕnn , ϕnp and ϕpp .

2 Input

It is reasonable to consider a 3- particle system of the type $\phi\phi N$ in the same theoretical frameworks. To describe this system, one needs apart from the $\phi - N$ potential also the $\phi - \phi$ potential. One of the simplest ways to construct this potential is to take it in the form of a sum of two Yukawa potentials (attractive and repulsive):

$$V_{\phi\phi}(r) = V_1 \frac{e^{-\mu_1 r}}{r} - V_2 \frac{e^{-\mu_2 r}}{r} \quad (1)$$

where

$$\begin{aligned} V_1 &= 1000 \text{ MeV} \cdot \text{fm} , \quad \mu_1 = 2.5 \text{ fm}^{-1} , \\ V_2 &= 1250 \text{ MeV} \cdot \text{fm} , \quad \mu_2 = 3.0 \text{ fm}^{-1} . \end{aligned}$$

The parameters of this potential were fixed by the position and width of the f2(2010) resonance which has only one decay channel into two ϕ - mesons.

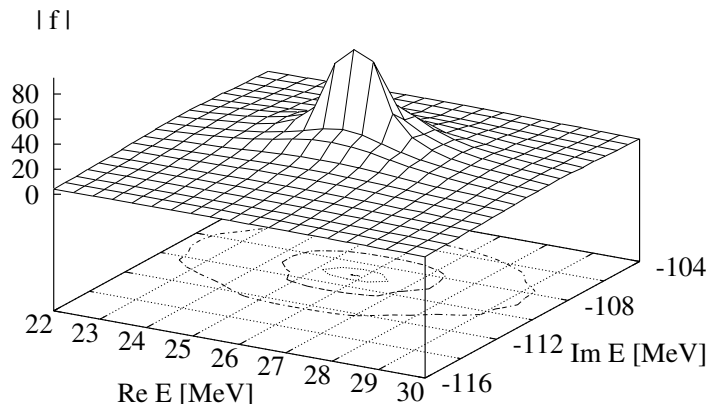


Figure 2: The location of the pole of the absolute value of the $\phi-\phi$ scattering amplitude in the complex energy plain.

As in [9], a Yukawa type potential is chosen for the $\phi - N$ interaction

$$V_{\phi N}(r) = -\alpha e^{-\mu r}/r \quad (2)$$

with $\alpha = 1.25$ and $\mu = 600$ MeV. This potential is rather deep and narrow and supports binding in the ϕN system with binding energies $E_{\phi n} = -9.47$ and $E_{\phi p} = -9.40$ MeV.

For the np triplet s-wave interaction the potential MT III [10] was used. Our singlet s-wave interaction is based on the potential MT I [10] with a slight modification of having now a parameter $\lambda_A = 2.617$. This value is chosen in order to reproduce the experimental value of the nn -scattering length $a_{nn} = -18.5$ fm [11]. One can see that in the three-body systems ϕNN there are two scales of distance related to the different ranges of the $N-N$ and $\phi-N$ interactions. This may produce a delicate interplay between a narrow attraction area of the $\phi - N$ interaction and repulsive parts of the MT-potentials, as it was emphasized in [12]. Apart from that, different ranges of the interaction can provide the cluster formation in the systems under consideration.

Let us start to describe three particle systems.

3 Calculations

Our calculations are based on Faddeev equations [13] in a differential form [14] written down for the 3-body systems ϕNN and $\phi\phi N$.

First, the Faddeev components of the wave function are expanded into partial waves:

$$\begin{aligned} \Psi_\alpha(\vec{\eta}_\alpha, \vec{\xi}_\alpha) = & \sum_{LMl\lambda} \frac{1}{\eta_\alpha \xi_\alpha} U_{\alpha l \lambda}^L(\eta_\alpha, \xi_\alpha) Y_{l\lambda}^{LM}(\hat{\eta}_\alpha, \hat{\xi}_\alpha) \stackrel{L=l=\lambda=0}{=} \\ & \frac{1}{\eta_\alpha \xi_\alpha} U_\alpha(\eta_\alpha, \xi_\alpha) Y_{00}^{00}(\hat{\eta}_\alpha, \hat{\xi}_\alpha) \end{aligned} \quad (3)$$

where $\eta_\alpha = |\vec{\eta}_\alpha|$, $\xi_\alpha = |\vec{\xi}_\alpha|$, $\hat{\eta}_\alpha = \vec{\eta}_\alpha/|\vec{\eta}_\alpha|$, $\hat{\xi}_\alpha = \vec{\xi}_\alpha/|\vec{\xi}_\alpha|$, $Y_{l\lambda}^{LM}$ are the bispherical harmonics. Jacobi coordinates $\vec{\eta}_\alpha, \vec{\xi}_\alpha$ were used, and only the lowest partial wave is taken into account.

The Jacobi coordinates are as usual:

$$\vec{r}_i - \vec{r}_j = \frac{\vec{\eta}_\alpha}{a_\alpha} \quad (4)$$

$$\frac{m_i \vec{r}_i + m_j \vec{r}_j}{m_i + m_j} - \vec{r}_k = \frac{\vec{\xi}_\alpha}{b_\alpha} \quad (5)$$

where \vec{r}_i , m_i denote the radius-vector and the mass of particle i ,

$$a_\alpha = \sqrt{\frac{m_i m_j}{(m_i + m_j) M}}, \quad b_\alpha = \sqrt{\frac{m_k(m_i + m_j)}{M^2}}, \quad M = m_1 + m_2 + m_3$$

and indices α take on the following values: $\alpha = 3$ for $(ij)k = (12)3$, $\alpha = 1$ for $(ij)k = (23)1$, $\alpha = 2$ for $(ij)k = (31)2$.

Since there are two identical particles in the system (we take $m_N = m_n$ for ϕnp system), the following two coupled-differential Faddeev equations survive:

$$\left\{ \begin{aligned} \left[\widehat{D} + V_1\left(\frac{\rho \cos \varphi}{a_1}\right) - E \right] U_1(\rho, \varphi) = & -V_1\left(\frac{\rho \cos \varphi}{a_1}\right) \sum_{\alpha' \neq 1} \frac{1}{\sin(2\gamma_{\alpha'1})} \int_{c-}^{c+} U_{\alpha'}(\rho, \varphi') d\varphi' \\ \left[\widehat{D} + V_2\left(\frac{\rho \cos \varphi}{a_2}\right) - E \right] U_2(\rho, \varphi) = & -V_2\left(\frac{\rho \cos \varphi}{a_2}\right) \sum_{\alpha' \neq 2} \frac{1}{\sin(2\gamma_{\alpha'2})} \int_{c-}^{c+} U_{\alpha'}(\rho, \varphi') d\varphi' \end{aligned} \right. \quad (6)$$

$$(U_3 \equiv U_2)$$

where polar coordinates $\rho = \sqrt{\eta_\alpha^2 + \xi_\alpha^2}$, $\tan \varphi_\alpha = \xi_\alpha/\eta_\alpha$ are introduced and

$$\begin{aligned} V_1 &= V_{NN}, \quad V_2 = V_{\phi N}, \\ \widehat{D} &= -\frac{\hbar^2}{2M} \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} \right) \\ c+ &= \text{Min} \{ |\varphi + \gamma_{\alpha'\alpha}|, \pi - (\varphi + \gamma_{\alpha'\alpha}) \}, \quad c- = |\varphi - \gamma_{\alpha'\alpha}| \\ \gamma_{ij} &= \arcsin s_{ij}, \quad s_{ij} = \sqrt{\frac{m_k M}{(m_i + m_k)(m_j + m_k)}}, \\ (ijk &= 123, 231, 312) \end{aligned}$$

indices correspond to 1 for ϕ -meson, 2 and 3 for nucleons.

The two-dimensional system of Faddeev equations (6) has been solved by discretization of variables: hyperradius ρ and hyperangle φ with N and M mesh points, respectively. Stable results for three digits of binding energies were reached at $N = 110$, $M = 210$ and $L(\rho \text{ variable cutoff}) = 9 \text{ fm}$.

As a result, the binding energy of the system ϕnn with value $E_{\phi nn} = -21.8 \text{ MeV}$ and value $E_{\phi np} = -37.9 \text{ MeV}$ for the binding of the ϕnp system with np pair in triplet state have been obtained. It should be noticed that for this binding energy in the ϕnp system both the main ϕ -meson decay channels on K -mesons are closed. Let us comment the last value of energy which appeared rather large. From naive reasons in the configuration $\phi + d$ one would expect binding of an order of $2 \times E_{\phi N} + E_d$ which is much smaller than the calculated value. However, due to the strong attraction in the ϕN - subsystem ($E_{\phi N} \sim -9 \text{ MeV}$) one can expect that in the 3-particle ϕnp system the configuration $\phi + d$ is rather suppressed. Hence, it follows that in the above system there is no strong cancellation between potential and kinetic energies of nucleons, like in deuteron, and a strong attractive triplet $N - N$ potential ($V_t \sim 100 \text{ MeV}$) shows its full value.

The dependence of the ϕnn binding energy on the parameter α of the $\phi - N$ interaction is investigated. It is shown in Figure 3 that excited states appear in this system.

As can be seen from these results, the binding in 3-particle systems like ϕNN is possible even for weaker $\phi - N$ attraction than the one of potential (2).

Now let us turn to the $\phi\phi N$ system. The indication of a resonance in the $\phi + \phi$ subsystem and its parameters have been found by drawing the Argand diagram and studying the position of the pole in the scattering amplitude.

With the potential $V_{\phi\phi}$ of the form (1), the binding energy of the 3-body system $\phi\phi n$ turns out to be - 77 MeV.

As in the case of the $\phi(np)_{\text{triplet}}$ state, due to the strong binding in the $\phi\phi N$ system, the K-meson decay channels of the ϕ - meson are closed.

Let us discuss now the 4-body system ϕnnn . To make a preliminary estimate of its binding energy, we have used the folding model with the (ϕnn) -subsystem as a cluster.

By averaging the interactions of the third neutron with the particles of the (ϕnn) system over the cluster wavefunction it is easy to obtain an effective potential which has the form shown in Figure 4. Here r means the distance between the third neutron and the center of mass of the (ϕnn) cluster.

The solution of the Schroedinger equation with this potential shows that there are no bound states. However, the exact treatment of an analogous 4-body system $(\eta_c + 3N)$, performed by means of the AGS- equations [15], shows that the folding model may strongly underestimate the exact calculation.

We plan to perform such a calculation in the near future.

Finally, let us discuss the matter distribution, shown in Figures 5 and 6, for the ϕnn system. First of all, as one can see on both the pictures which show the profiles of two Faddeev components of the wave function, the main part of a matter is concentrated at small distances inside the volume of the size of 2 fm. In addition, one can see that the fine structure of different components of the wave function is nonhomogeneous. Indeed, the component U_2 has an extremum at hyperangle $\varphi \sim \pi/2$ which means that a ϕ - meson and one of neutrons are close to each other forming the substructure in the ϕnn system, which was mentioned at the end of the section 2. In contrast to that, the component of the wave function U_1 has two extrema without well- pronounced substructures. Having in mind the small size of the system and its neutrality, one would expect the propagation of an object like this far away from the region of production.

Acknowledgments

The work was supported by the Deutsche Forschungsgemeinschaft (DFG grant no 436 RUS 113/761/0-2) and the Heisenberg-Landau Program 2008.

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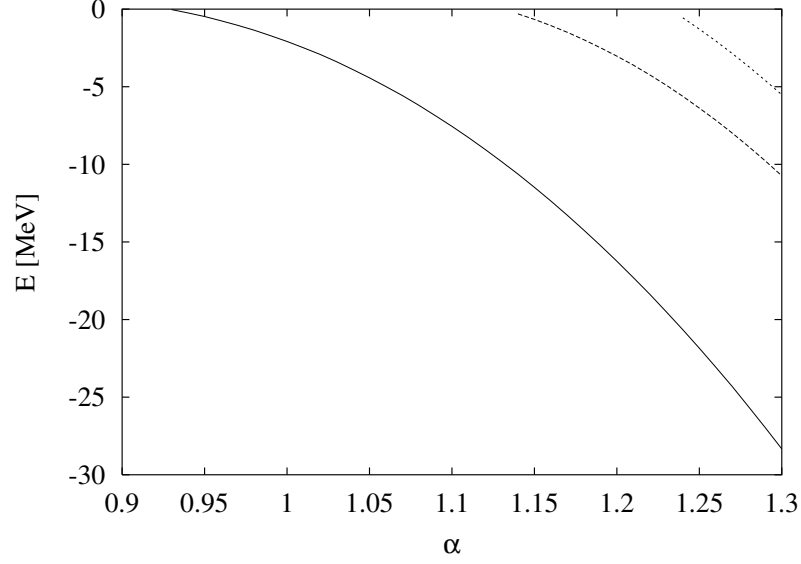


Figure 3: The dependence of the binding energy of the ϕnn system on the parameter α of the $\phi - N$ interaction.

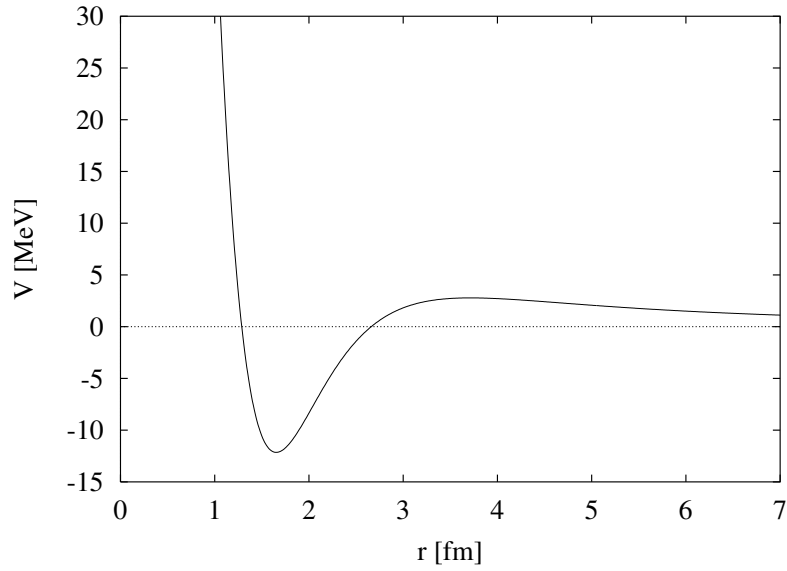


Figure 4: The folding potential with the p-wave centrifugal barrier for the four-body system $(\phi nn) + n$.

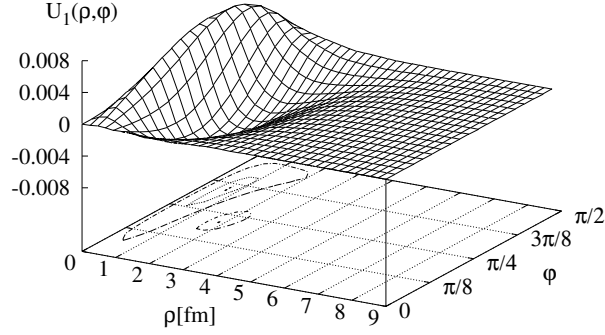


Figure 5: The partial amplitude $U_1(\rho, \varphi)$ for the ϕnn system. Extreme values of $U_1(\rho, \varphi)$ are 6.772×10^{-3} (at $\rho = 0.654$, $\varphi = 1.092$) and -5.633×10^{-3} (at $\rho = 1.881$, $\varphi = 0.695$). Contours of different cross-sections of the $U_1(\rho, \varphi)$ surface are shown in the (ρ, φ) plain. Dotted, dashed-dotted and second dashed-dotted lines correspond to values 5.643×10^{-3} , 1.693×10^{-3} , -5.121×10^{-3} on the $U_1(\rho, \varphi)$ axis respectively.

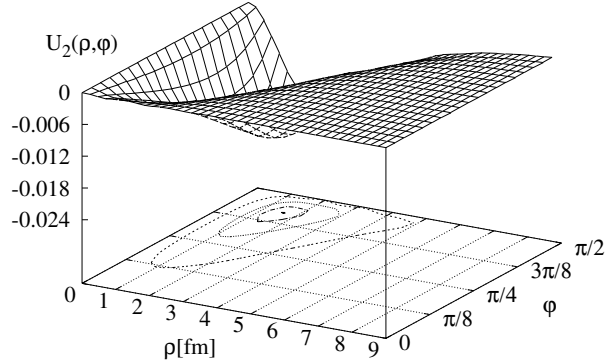


Figure 6: The partial amplitude $U_2(\rho, \varphi)$ for the ϕnn system. The extreme value of $U_2(\rho, \varphi)$ is -2.182×10^{-2} (at $\rho = 1.554$, $\varphi = 1.323$). Contours of different cross-sections of the $U_2(\rho, \varphi)$ surface are shown in the (ρ, φ) plain. Dashed-dotted, dotted and dashed lines correspond to values -1.983×10^{-2} , -1.454×10^{-2} and -0.545×10^{-2} on the $U_2(\rho, \varphi)$ axis respectively.